

# ME41 Project 2 - Analysis of Machine Elements

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## Executive Summary

Our group was given the choice to examine either the drill press or injection molder. We decided to examine the drill press, due to its relative complexity and our own personal interest.

The drill press consists of many different machine elements working together to deliver power from the motor to the drill bit. We decided to focus on the belt and shaft (spindle) elements specifically, which are two critical components in the power delivery mechanism. A *belt* is a system of *pulleys* that is responsible for both transferring power from the motor to the spindle, and also varying the speed of the shaft through different pulley combinations. A belt drive has many advantages over a geared transmission in applications where precise speed control isn't paramount since it requires less maintenance/is easier to replace, is cheaper, and is less noisy, among others. The *spindle* is the main spinning element of the drill press. It can rotate freely within the *quill*, which can translate vertically. The top portion of the *spindle* consists of *splines* which allow the shaft to spin in unison with the belt drive. The bottom portion interfaces with the *arbor*, which in turn holds the chuck and drill bit in place.



**Figure 1:** Left - Example belt and pulley system with top of spindle poking out of leftmost pulley.  
Right - Example chuck, spindle, and quill

**Table 1:** Analytical factors of safety for belt and spindle

	Factor of Safety
Belt Drive	<b>1.15</b>
Spindle	<b>3.7</b>

## Introduction

The efficiency and reliability of mechanical systems heavily rely on the structural integrity of their components, especially in machinery like drill presses where power transmission elements, such as belts and spindles, play a critical role. This project aims to conduct a comprehensive analysis of the belt and spindle components within a drill press utilizing both analytical and Finite Element Analysis (FEA) techniques. The primary objective is to ascertain the factors of safety associated with various failure modes of these elements, ultimately discerning the most probable manner in which a failure might occur. The designer can then use such information to make informed design decisions on what components, if any, should be strengthened. In general, these factors of safety can aid in proactive maintenance, design improvements, and potential redesign considerations.

## Belt Approach - Cardi Garcia Mendez

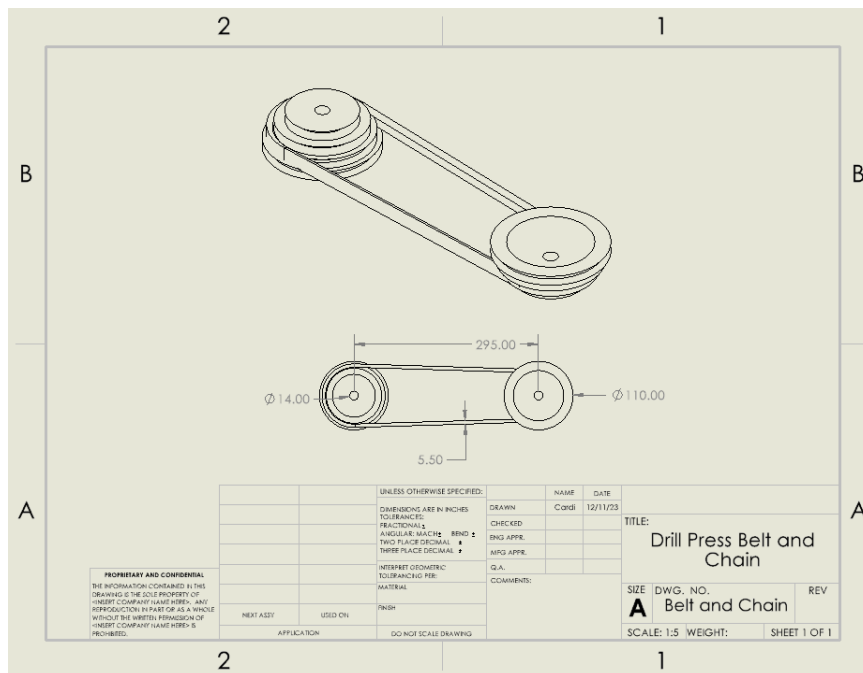
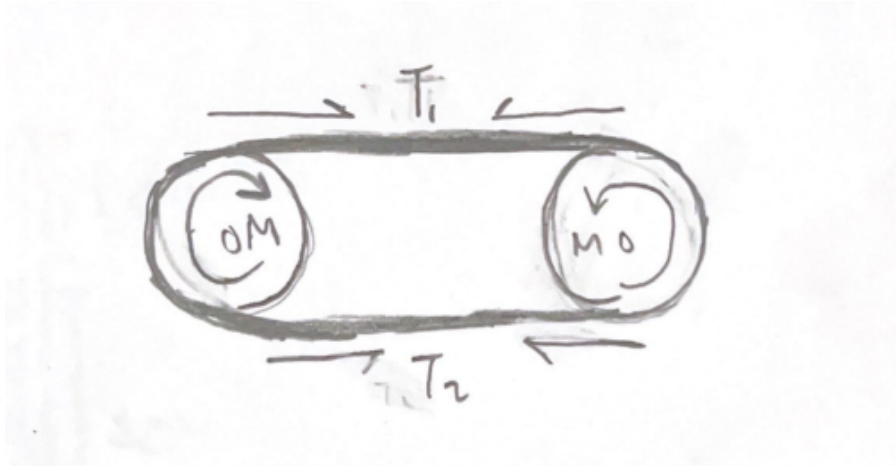


Figure 2: Belt and pulley engineering drawing

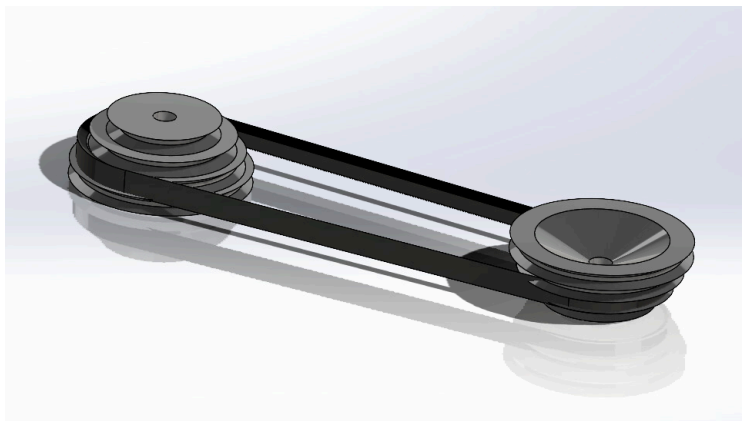


**Figure 3:** Belt and pulley FBD

## Belt Analysis - Cardi Garcia Mendez

### Assumptions

Some assumptions had to be made for the analysis of the belt and chain. The biggest assumption that had to be made was the materials used in the SolidWorks FEA. In most belt and chain designs plastics alloy such as Nylon or Ultra High Molecular Weight Polyethylene (UHMW) are used, so the assumption made for the FEA was to use a Nylon-101 material for the plastic used to make the gears and belt.

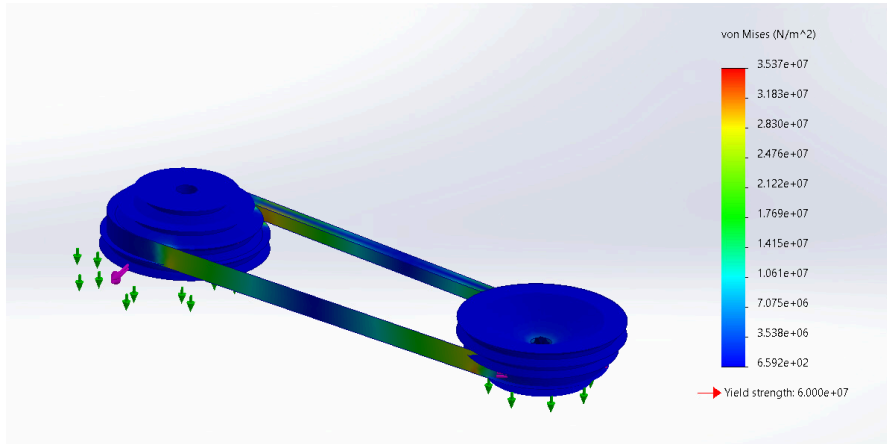


**Figure 4:** Cad model of belt drive

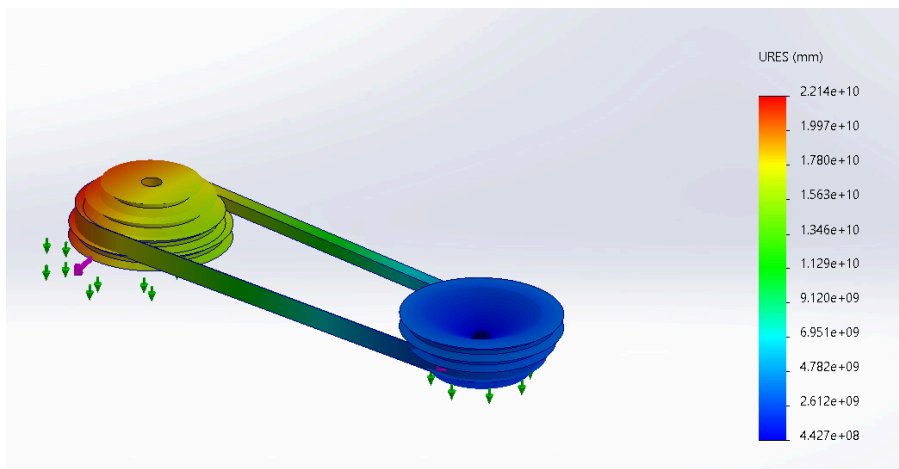
Material used was Nylon101 because of its common use in other belt drive systems.



## FEA



**Figure 5:** Yield prediction FEA



**Figure 6:** Displacement prediction FEA

For the simulation, a torque force was added to both gears at the slot where the belt was connected to.

## Analytical Analysis

I further analyzed the drill press's belt and chain components, by employing calculations to determine Torque, Tight Side Tension (F1), Loose Side Tension (F2), and subsequently, the factor of safety for the belt and chain.

Due to the absence of comprehensive information from the machine manufacturers some assumptions were necessary. For example, I had to estimate the *nominal power* of the belt to equal 6. As well as estimating that the specification of the belt had to be A-4 (*Polyamide*) since the thickness of the belt was 5.5mm or about .20 in and made of a Nylon material.

Torque Calculations:

$$n = 824 \text{ RPM at } 60 \text{ Hz } (n_d = 1.1)$$

$$K_s = 1.2 \text{ (short, normal use 8-10 hrs daily)}$$

$$T = \frac{63025 H_{nom} K_s n_d}{n} = \frac{(63025)(15)(1.2)(1.1)}{824}$$

$$T = 1514.43 \text{ lbf}\cdot\text{in}$$

Tight Side (F1) and Loose Side (F2) Tension Calculations:

$$d = 73 \text{ mm} \approx 2.87 \text{ in} \quad b = 5.5 \text{ mm} \approx 0.21 \text{ in}$$

$$D = 91 \text{ mm} \approx 3.58 \text{ in} \quad C = 29.5 \text{ cm} \approx 11.6 \text{ in}$$

$$(F_1 - F_2) \rightarrow \frac{2T}{d} = \frac{(2)(1514.43)}{2.87} = F_1 = 527.66 \text{ lbf}$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 527.66 - 36.75$$

$$F_2 = 490.91 \text{ lbf}$$

Factor of Safety Calculations:

$$V = \frac{\pi d n}{12} = \frac{\pi (2.87)(824)}{12} = \underline{619.12 \text{ ft/min}}$$
$$(F_1)_a = \frac{b F_a C_p C_r}{33000} = \frac{(.21)(175)(1)}{33000} = \underline{36.75 \text{ lbf}}$$

from table 17-2 ↙

$$H = \frac{(F_2 - F_1)V}{33000} = \frac{(385.39 - 36.75)}{33000} = 8.32$$
$$n_{fs} = \frac{H}{H_{nom} K_s} = \frac{8.32}{(6)(1.2)} = \boxed{n_{fs} = 1.15} \checkmark$$

A Safety Factor of 1.15 was calculated, which indicates a proper safety factor.

## Summary

In summary, the analytical Finite Element Analysis (FEA) of the belt and pulley system indicated that, when subjected to modest stresses and forces, the belt consistently maintains a satisfactory factor of safety. Furthermore, the comprehensive analytical and FEA analysis demonstrated that, even when exposed to comparatively high loading conditions, the spindle consistently upholds a reasonable factor of safety.

A static failure analysis of the belt and chain was meticulously conducted through FEA simulations, incorporating calculations for Torque, Tight Side Tension (F1), Loose Side Tension (F2), and resulting in a safety factor of 1.5.

# Spindle Approach - Alan Deutsch

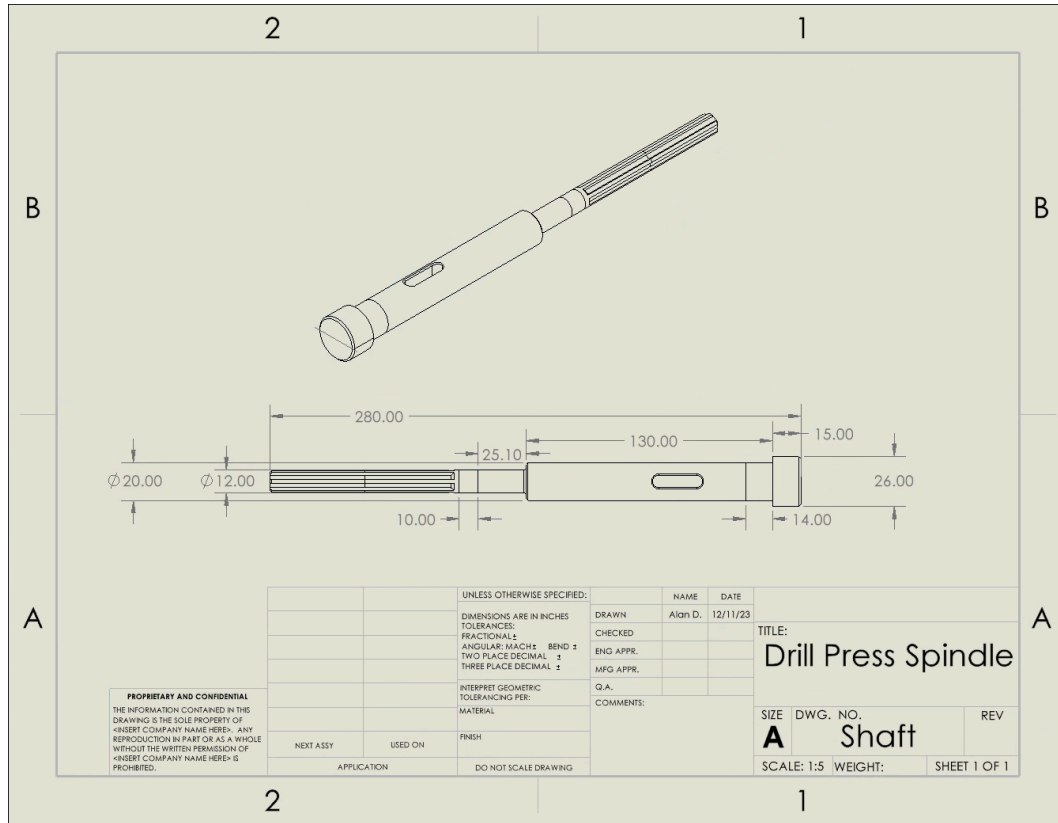


Figure 7: Engineering drawing of spindle

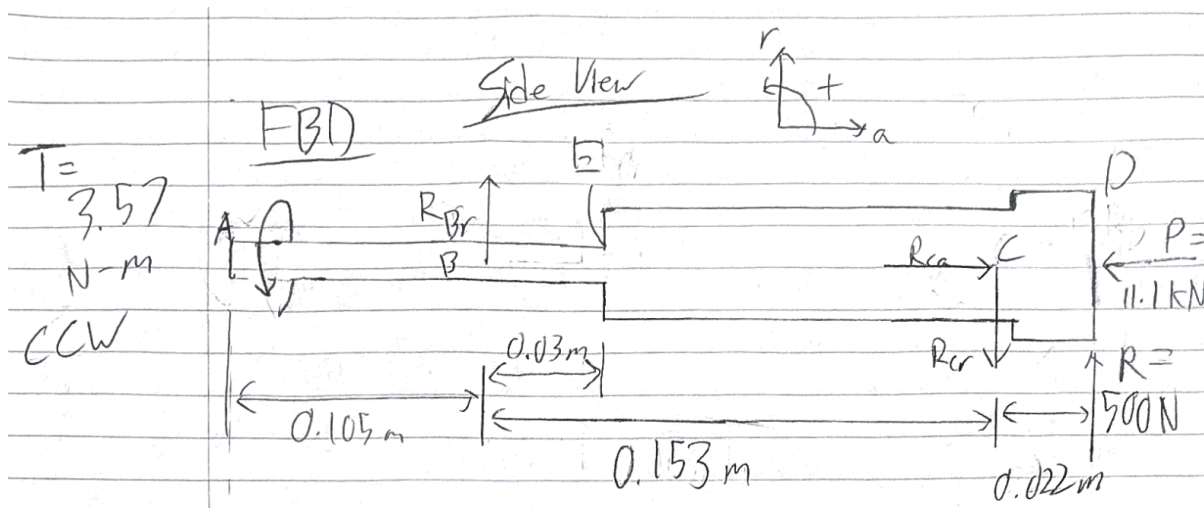


Figure 8: FBD Diagram of spindle showing applied forces and reactions

## Spindle Analysis - Alan Deutsch

### Assumptions

#### System Simplification

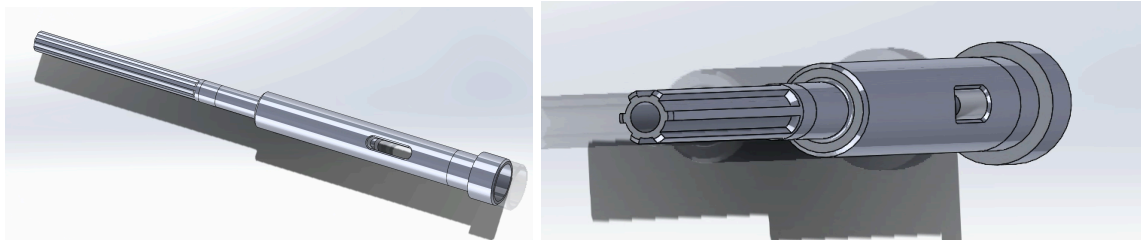
Though I was unable to disassemble the drill press in Bray to get a closer look at the spindle, I was able to find specifications of similar spindles online. It turns out that the basic drill press spindle design is very similar across many manufacturers. I based the overall dimensions of my model off of [this](#) particular eBay listing for a spindle from a “vintage Taiwan-made drill press.”



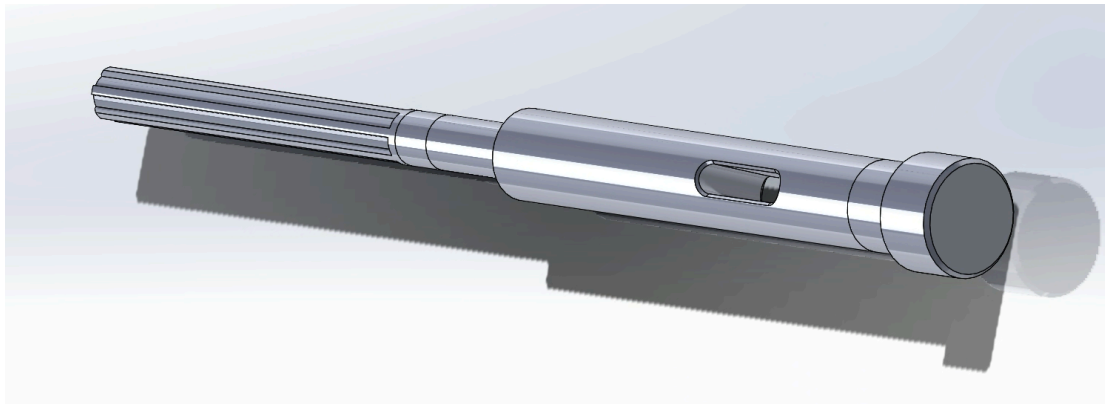
**Figure 9:** Quill/Spindle Assembly from eBay listing

Unfortunately, the spindle is partly hidden by the quill in the photos in the listing, so I needed to find other resources to confirm the dimensions of the spindle. Using my best judgment, I cross-checked what I could tell from the listing with various youtube [videos](#) of people disassembling/restoring their spindles and [instruction manuals](#) to come up with a satisfactory shape that fit within the overall dimensions from eBay. Since I was able to take measurements of the larger bearing in the quill/spindle assembly of the drill press in Bray, I was able to cross check my design with this as well. I think the final CAD design I came up with is a sound representation of a basic drill press spindle. It includes the splines used to interface with the belt drive as well as the slotted hole used for disassembly.

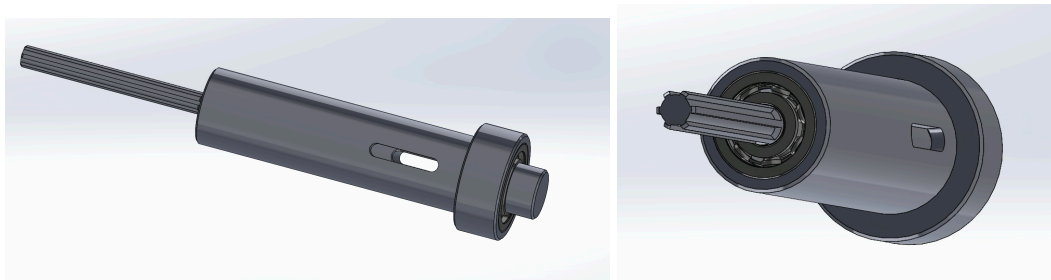
The eBay listing clearly shows that at least parts of the spindle are hollow. Since I wasn't sure of the spindle's exact internal dimensions and to simplify, I decided to make the spindle have a solid cross section throughout. For the analytical calculations, I decided to further simplify by not considering the splines nor the slot.



**Figure 10:** Views of original CAD design with hollow cross section



**Figure 11:** Modified solid-cross-section design



**Figure 12:** Entire quill/spindle assembly with bearings

Although I modeled the quill to get the overall dimensions right, it is not the focus of this project.

## Selection of Applied Forces

Deciding on appropriate values for the forces applied to the spindle was quite difficult. In order to consider a worst-case scenario and make the analysis most interesting, I knew the spindle would undergo a combined loading of torsion, compression, and bending. I then came up with forces that could arise from extreme misuse of the machine.

### Compression

For compression, I considered the force a 100 lb person could produce by placing their entire body weight on the handle of the machine. I calculated this by determining the mechanical advantage of the rack and pinion that causes the vertical translation of the quill assembly. In order to do this, I measured the linear vertical translation of the quill caused by a single revolution of the handle. I then divided the circumference of the circle traced by the handle in a full revolution by this linear distance to determine the force multiplier. Finally, I multiplied the force of 100 lbf by this value to get 2550 lbf applied to the spindle.

#### Calculations:

1 full revolution -> 50 mm of linear translation

Handle is 8 inches = 203.2 mm away from the axis of rotation

Arc of full revolution =  $203.2 * 2\pi = 1276.7$  mm

Force multiplier =  $1276.7 / 50 = 25.5$

Compressive force =  $100 * 25.5 = 2550$  lbf = **11.3 kN**

\*Before I realized I would be working with metric units I rounded 2550 lbf down to 2500 lbf, which I then converted to 11.1 kN which is the value I used in my calculations.

#### How its applied:

In the analytical model, it's a point force applied at the right-most edge of the FBD. In the FEA, it's distributed over the right-most face of the geometry.

### Torsion

For torsion, I initially tried to find the torques imparted on drill bits when drilling through various materials, but after much looking I wasn't able to find any satisfactory data. I settled on a worst-case scenario where the drill bit gets stuck in the material and the motor is applying close to its maximum output torque on the spindle.

#### Calculations:

Motor produces  $\frac{1}{2}$  HP (as shown on the side of the actual Bray drill press)

Assume 50% power is transmitted through to the shaft

Assume a drilling speed of 1000 RPM

Torque = HP \* 5252 / RPM = .5 \* 5252 / 1000 = 2.63 ft-lbf = **3.57 N-m**

#### **How its applied:**

For analytical and FEA it's a clockwise torque when viewed from above, applied at the spline end of the spindle. It's clockwise because the bit is driven clockwise, so it would experience a clockwise torque from above when it gets stuck.

#### Radial/Bending

Since drill presses are designed to be used in a purely up and down fashion, radial forces at the bit that you would encounter in mill operation are a clear form of misuse. Since we are trying to formulate a worst-case scenario, we will assume a 50 kg person is able to push their entire weight onto the workpiece (**500 N**), which is fully transferred to the spindle. We will assume the 500 N is applied directly at the end of the spindle instead of at a distance away to simplify calculations.

I could have also considered the radial force due to the pulley at the top of the shaft, which is a more realistic condition than this scenario, but I thought it would be more interesting to focus on how potential misuse of the machine could affect the likelihood of failure.

#### **How its applied:**

For analytical and FEA it's applied at the wider end of the spindle. For analytical it's a point force and for FEA it's distributed over a portion of the outer cylindrical face.

### Material Selection and Properties

For the material of the spindle I decided on **AISI 4140 steel**, as multiple sources online claimed it is a common material for drill press spindles, and is in general an ideal material for shafts undergoing cyclic loading.

Elastic Modulus: 200 GPa

Tensile Strength: 665 MPa

Yield Strength: 415 MPa

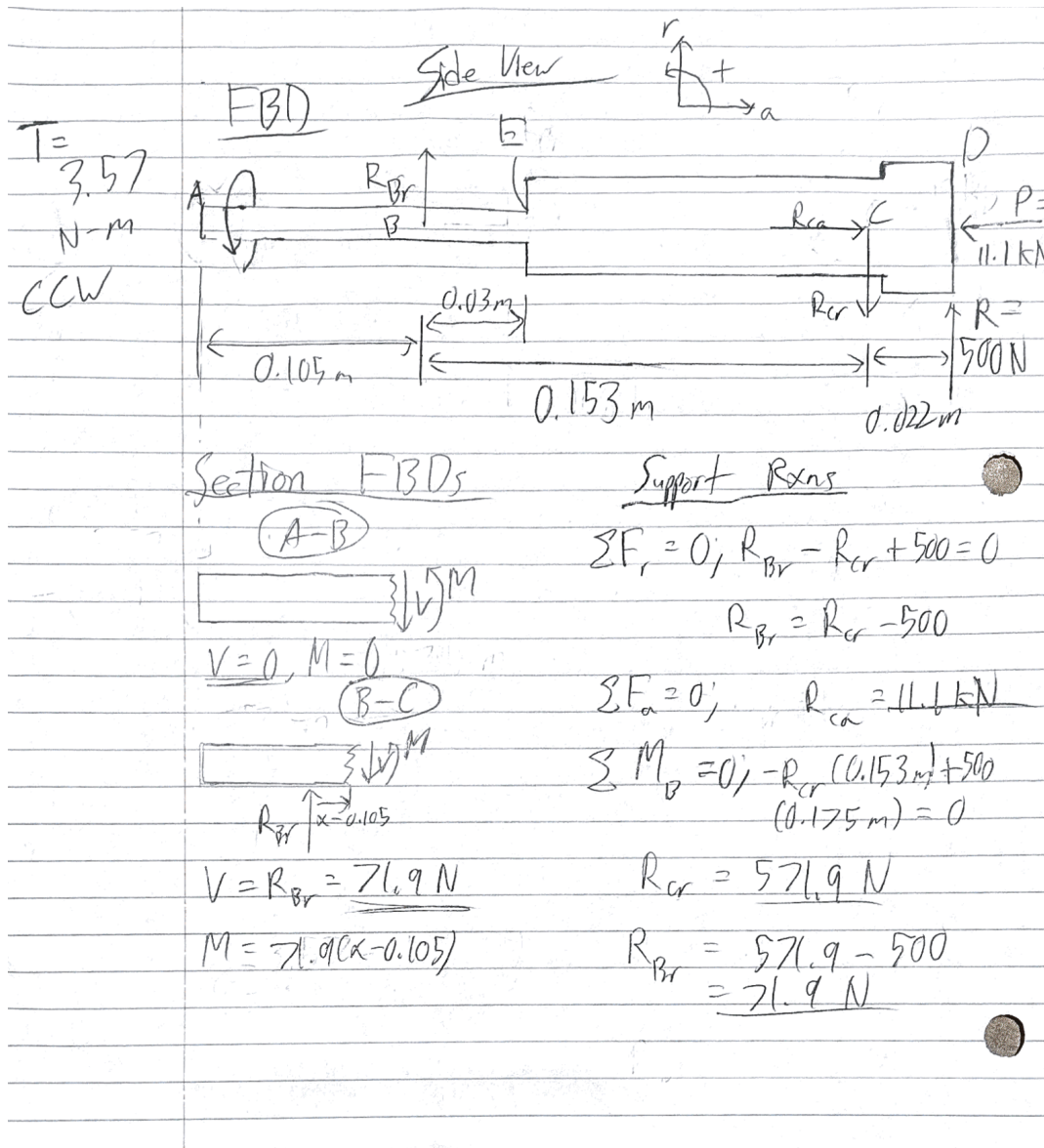
### Looking Ahead

In the next section we will look at the static and fatigue analytical analyses and static and fatigue FEA analyses, in that order. I opted to discuss FEA after analytical, because it makes more sense to me to have it as a verification of analytical results. I decided to consider both static and fatigue failure because they are both potential failure modes for such a machine element.



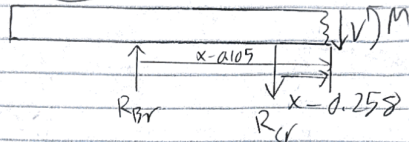
## Analytical Analysis - Static

The static analysis of the spindle continues off of the FBD shown in the approach section. First we find the support reactions at the two bearings. Then we can begin drawing the section FBDs to work towards drawing the internal shear and moment diagrams.



Section FBDs Continued

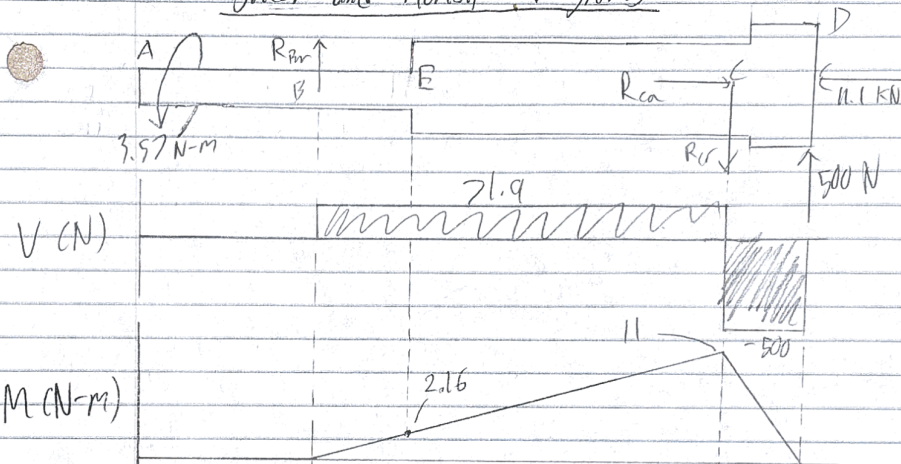
(C-D)



$$V = R_{Br} - R_{Cr} = 71.9 - 571.9 = -500 \text{ N}$$

$$M = 71.9(x - 0.105) - 571.9(x - 0.258)$$

Shear and Moment Diagrams



From the shear and moment diagrams we can see the areas in the spindle most at risk of failure. Although the highest moment and shear technically occurs at the second bearing (C), the diameter there is large and the surface is supported by the bearing. Thus, the most likely candidate for failure is point E, at the top of the spindle where the smallest diameter section meets the middle diameter section. The surface here experiences both shear from the torque and moment from the radial force, and because of the aggressive change in diameter, a stress concentration will occur here which will worsen fatigue, although this won't affect static failure

because of local strain hardening in ductile materials. Thus, an element at point E will be the focus of both our static and fatigue analysis.

Our first step in the analysis of point E is to calculate the values of all the stresses acting on the element.

● Static Analysis - Point E

Consider element at top of shaft, point E.

① Assume solid, 12mm dia. bar.

Torsion shear  $\rightarrow \tau_T = \frac{Tr}{J} = \frac{Tr}{\frac{\pi}{2}r^4}$

$$J = \frac{\pi}{2}r^4 = \frac{2T}{\pi r^3}$$

$$= \frac{2(3.57 \text{ N}\cdot\text{m})}{\pi(0.006 \text{ m})^3}$$

$$= 10.5 \text{ MPa}$$

Pure shear  $\rightarrow \tau_s = \frac{R}{A} = \frac{29 \text{ N}}{\pi(0.006 \text{ m})^2} = 0.636 \text{ MPa}$   
 (zero on outer surface)

Compressive stress  $\rightarrow \sigma_c = \frac{P}{A} = \frac{11.1 \text{ kN}}{\pi(0.006 \text{ m})^2} = 98.1 \text{ MPa}$

Bending stress  $\rightarrow \sigma_b = \frac{Mc}{I} = \frac{32 \text{ M}}{\pi D^3} = \frac{32(2.16 \text{ N}\cdot\text{m})}{\pi(0.012 \text{ m})^3}$

② Find stress coordinates  $= 12.7 \text{ MPa}$

It is worth noting that in this case the pure/transverse shear caused by the radial force doesn't contribute any stress to the element because transverse shear is zero on the outer surfaces of a member.

Next we can write down the stress coordinates and draw Mohr's circle for the element.

② Find stress coordinates  $= 12.7 \text{ MPa}$

X-axis  
 $\sigma_x = \sigma_C + \sigma_B = (98.1 + 12.7) = -110.8 \text{ (compression)}$

$\tau_{xy} = \tau_T = 10.5 \text{ MPa CCW}$

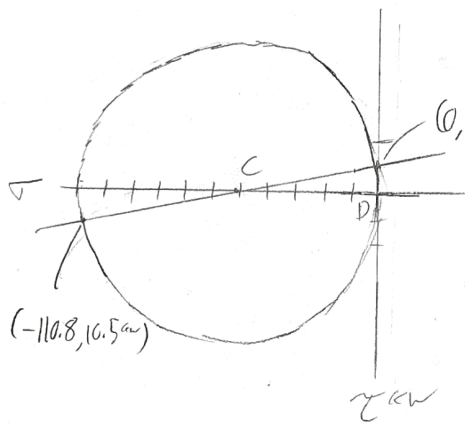
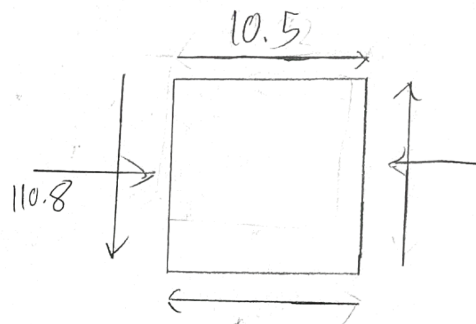
Y-axis  
 $\sigma_y = 0$

Static Analysis Cont.

State of Stress:

③

Mohr's Circle:



$$C = \frac{\sigma_x + \sigma_y}{2} = -55.9$$

$$CD = 55.9$$

$$R = \sqrt{55.9^2 + 10.5^2} = 56.4 \text{ MPa}$$

$$\sigma_1 = C - R = -112 \text{ MPa}, \quad \sigma_2 = C + R = 0.986 \text{ MPa}$$

$$\tau_{\text{max}} = 56.4 \text{ MPa}$$

Mohr's circle ↓

From Mohr's circle we know the two principal stresses and the max shear stress acting on the element. We can then apply the Rankine failure criterion, which states that failure occurs simply when either of the principal stresses equals the tensile yield stress and/or max shear equals the shear yield stress, to obtain our first factors of safety for failure in compression and shear.

For 4041 steel,  $S_y = 415 \text{ MPa}$ ,  $S_{sy} = .5 S_y = 207.5 \text{ MPa}$

④ Rankine Failure:  $n_y = \frac{S_y}{\tau_{max}} = \frac{-415}{-112} = \boxed{3.7}$

\* Assume  $S_{yt} = S_{yc}$   $n_{sy} = \frac{S_{sy}}{\tau_{max}} = \frac{207.5}{56.4} = \boxed{3.7}$

Thus, according to this failure theory, failure is equally likely to occur in compression as in shear.

However, this failure theory is inaccurate for ductile materials, since it doesn't consider that ductile materials don't readily yield under hydrostatic loading. We will thus apply the Distortion Energy (von Mises) failure theory to conclude our static analysis.

⑤ Distortion-Energy (DE) Theory:

$$\begin{aligned} \sigma' &= (\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2} \\ &= ((110.8)^2 - 0 + 0^2 + 3(10.5)^2)^{1/2} \\ &= 112.3 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \sigma' &= \frac{S_y}{n} \rightarrow n = \frac{S_y}{\sigma'} \\ &= \frac{415}{112.3} \\ &= \boxed{3.7} \end{aligned}$$

\* Note we didn't consider stress concentration due to local strain hardening w/ ductile material.

We also obtain a factor of safety of 3.7, so we can be confident that the static condition factor of safety is 3.7.

## Analytical Analysis - Fatigue

We will now consider the spindle under fatigue/cyclic loading. We can use almost the same loading conditions as those used in the static analysis, with some modifications to reflect a fatigue condition. In the static analysis, the scenario is indeed static as the spindle is stuck in whatever its drilling and thus isn't rotating. For a fatigue analysis, we obviously need to bring rotation into the equation \*pun intended\*. In order to create a scenario where the spindle is indeed rotating, we will lower the torque applied to the spindle by a factor of four, which should allow the spindle to rotate while still imparting significant torque. Secondly, it is unlikely that the bit is able to spin while a 100 lbf person is compressing the bit/spindle with their entire body weight times 25. In order to make the compressive force more reasonable for a fatigue setting, we will reduce it by a factor of 10. We will keep the 500 N radial force the same, although this is also likely much higher than what's possible in a fatigue setting.

### Fatigue Analysis

For fatigue analysis, we want to consider the spindle as it undergoes completely reversed bending and constant torsion and compression,  $(T_a = 0, M_m = 0, \sigma_{axial} = 0)$

Since the shaft would not be rotating if it were experiencing the full torque of the motor, we will lower the torque to roughly  $\frac{1}{4}$  of the value we used in the static analysis, which is still quite high  $\rightarrow T_m = 0.893 \text{ N}\cdot\text{m}$

Similarly, we will lower the compression force to represent more normal conditions  $\rightarrow P_m = \frac{11.1}{10}$

For AISI 4140 steel,  $\underline{\quad\quad\quad} = 1.11 \text{ kN}$



Our first step in the fatigue analysis is calculating the endurance limit. We do this by finding the endurance limit of a rotating beam specimen made of the same material, then applying Marin factors to find the endurance limit of the spindle.

For AISC 4140 steel,  $S_y = 415 \text{ MPa}$ ,  $S_{ut} = 655 \text{ MPa}$   $\overset{10}{=} 1.11 \text{ kN}$

Endurance Limit

$$S_e' = 0.5 S_{ut} = 327.5 \text{ MPa}$$

Table 6-2

Assume machined surface finish  $\rightarrow k_a = a S_{ut}^b$   $\downarrow$

$$= 4.51 (655)^{-0.265}$$
$$= \underline{0.809}$$

Fatigue Analysis Cont.

According to p. 326, use  $k_b, k_c$  for bending:

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{12 \text{ mm}}{7.62}\right)^{-0.107}, 2.79 \leq d \leq 51 \text{ mm}$$
$$= \underline{0.953}$$

The remaining Marin factors are all unity.

$$\begin{aligned} S_e &= k_a k_b k_c S_e' \\ &= (0.809)(0.953)(327.5 \text{ MPa}) \\ &= \underline{252 \text{ MPa}} \end{aligned}$$

The next step is finding the stress concentration factor due to the transition from small to large diameter, assuming a 1-mm radius fillet. We start by finding the theoretical stress concentration factors due to compression, torsion, and bending.

Since this is a fatigue analysis, we need to calculate stress concentration factors. Element E is located at the transition from the  $d=12\text{mm}$  section to the  $D=20\text{mm}$  section. We can assume the fillet is 1mm radius. Figures A-15-7, 8, 9 give us the theoretical stress concentration factors for this scenario.

$$D/d = \frac{20}{12} = 1.666, \quad r/d = \frac{1}{12} = 0.083$$

Compression:  $K_{t, \text{compression}} = 2.1$ , Torsion:  $K_{ts} = 1.5$ ,

Bending:  $K_{t, \text{bending}} = 1.8$

Because we are dealing with fatigue, the effect of the stress concentration on the development of cracks will be less than the theoretical values, which is accounted for by the notch sensitivity.

We need the notch sensitivities:

Notch radius = 1mm  $\rightarrow S_{ut} = 655\text{MPa}$

$$q_{\text{bending}} = 0.75, \quad q_{\text{shear}} = 0.8, \quad q_{\text{axial}} = 1$$

↑  
 We aren't given  $q$  for constant axial loading.

$$K_{f, \text{bending}} = 1 + q_{\text{bending}}(K_t - 1)$$

$$= 1 + 0.75(1.8 - 1)$$

$$= \underline{1.6}$$

$$K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

$$= 1 + 0.8(1.5 - 1) = \underline{1.4}$$

$$K_{f, \text{compression}} = K_t = \underline{2.1}$$



Now that we have the fatigue stress concentration values, we can apply them to the stresses due to bending, torsion, and compression. Compression and torsion are constant and thus midrange stresses, while bending is an alternating stress.

Alternating and Mean Stresses:

$$\begin{aligned}
 (\sigma_m)_{\text{comp}} &= K_{f,\text{comp}} \sigma_0 = 2.1 \left( \frac{P_m}{A} \right) = 2.1 \left( \frac{1.11 \times 10^3 \text{ N}}{\pi (0.006 \text{ m})^2} \right) \\
 &= 20.6 \text{ MPa} \\
 (\tau_m)_{\text{torsion}} &= K_{fs} \tau_0 = K_{fs} \frac{T (\frac{d}{2})}{\pi d^3 / 32} = K_{fs} \frac{16T}{\pi d^3} \\
 (\sigma_a)_{\text{bending}} &= K_{f,\text{bend}} \sigma_0 = 1.9 \left( \frac{16(0.893 \text{ N}\cdot\text{m})}{\pi (0.012 \text{ m})^3} \right) \\
 &= 3.68 \text{ MPa} \\
 \text{Same as before} &= K_f \left( \frac{Mc}{I} \right) = (1.6)(12.7 \text{ MPa}) \\
 &= \underline{20.4 \text{ MPa}}
 \end{aligned}$$

We can plug these stresses into the von Mises stress equations to find the alternating and midrange von Mises stresses on the element.

Now we can find the mean and alternating von Mises stresses:

$$\begin{aligned}
 \text{From P9.326} \left\{ \begin{aligned}
 \sigma'_a &= \left\{ \left[ (\sigma_a)_{\text{bending}} + \frac{(\sigma_0)_{\text{axial}}}{0.85} \right]^2 + 3 \left( (\tau_0)_{\text{torsion}} \right)^2 \right\}^{1/2} \\
 &= \left\{ (20.4)^2 \right\}^{1/2} \\
 &= \underline{20.4 \text{ MPa}} \\
 \sigma'_m &= \left\{ \left[ (\sigma_m)_{\text{bend}} + (\sigma_m)_{\text{axial}} \right]^2 + 3 \left( (\tau_m)_{\text{torsion}} \right)^2 \right\}^{1/2} \\
 &= \left\{ (20.6)^2 + 3(3.68)^2 \right\}^{1/2} \\
 &= \underline{21.6 \text{ MPa}}
 \end{aligned} \right.
 \end{aligned}$$

- best for ductile materials

Finally we can use the Gerber and Langer failure criteria to calculate the fatigue and first-cycle yielding factors of safety.

Use the Gerber and Langer failure criteria: 21.0 MPa  
 Good for ductile materials

Fatigue factor of safety:

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n(20.4)}{252} + \left(\frac{n(21.6)}{655}\right)^2 = 1 \rightarrow \boxed{\eta_f = 10.8}$$

First-cycle yielding:  $\eta_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{415}{20.4 + 21.6} = \boxed{\eta_y = 9.88}$

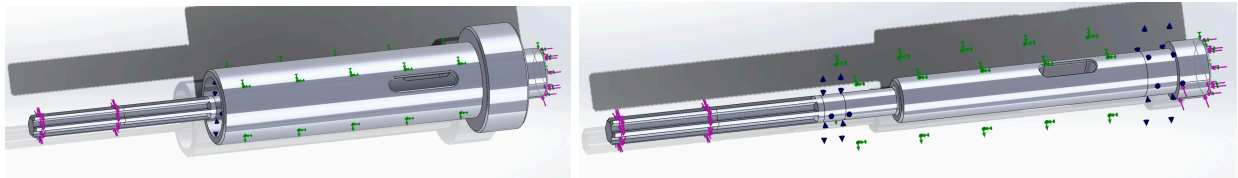
Fatigue factor of safety = **10.8**  
 First-cycle yielding factor of safety - **9.88**

As we can see, first-cycle yielding is more likely to occur before fatigue failure, although both factors of safety are very high and thus highly unlikely to occur.

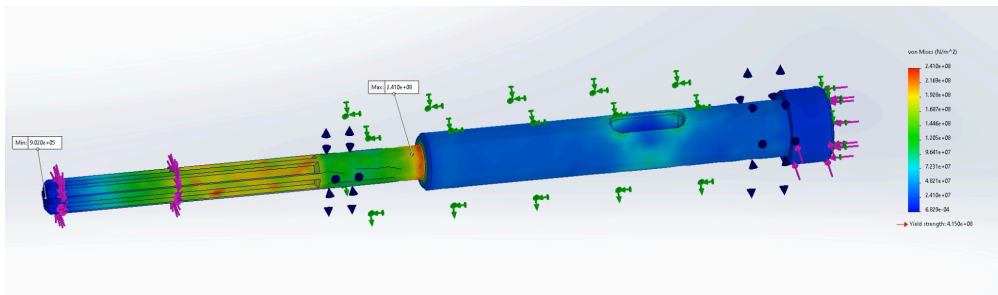
## FEA - Static

### Notes

Like the analytical analysis, this FEA considers compression, torsion, and bending all acting at once. Compression is provided by a force normal to the right-most face. Torsion is provided by a torque applied over a portion of the spline section of the spindle. Bending is provided by a force acting on a portion of the widest diameter section. The two bearings are represented by bearing connectors, and the quill is used as the single fixture. The second, larger bearing is seated against a shoulder inside the wider diameter part of the quill, which is what allows the spindle to resist compression. These boundary conditions are visible below.



**Figure 13:** Boundary conditions of FEA model

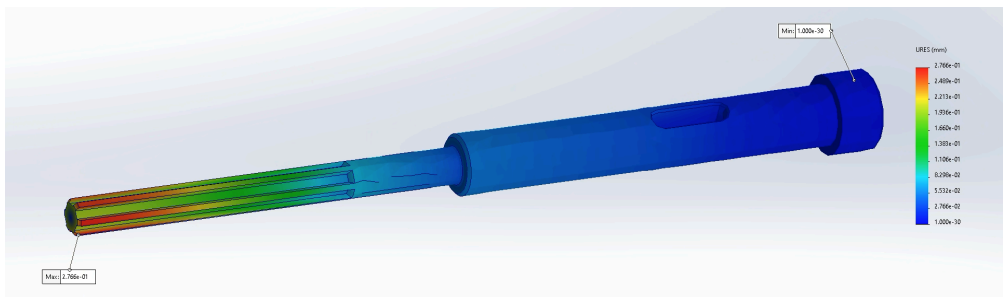


**Figure 14:** Von Mises Stress Plot

Max stress = **241 MPa**

Factor of safety =  $415 / 241 = 1.7$

This factor of safety is in the same order of magnitude as the analytical FOS.

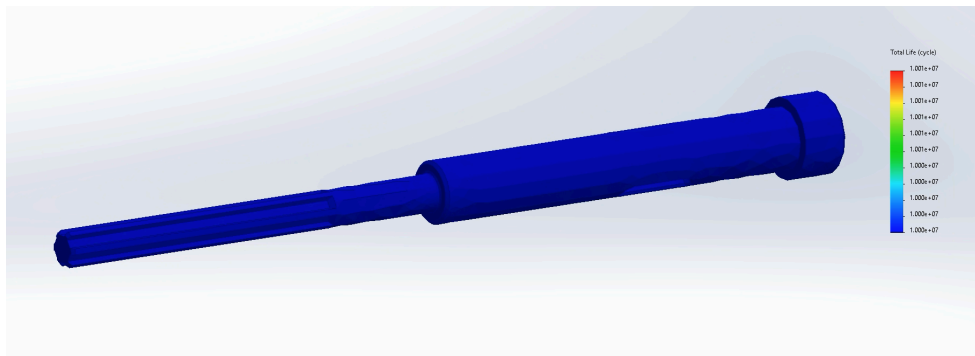


**Figure 15:** Displacement plot  
Max displacement = **0.277 mm**

## FEA - Fatigue

### Notes

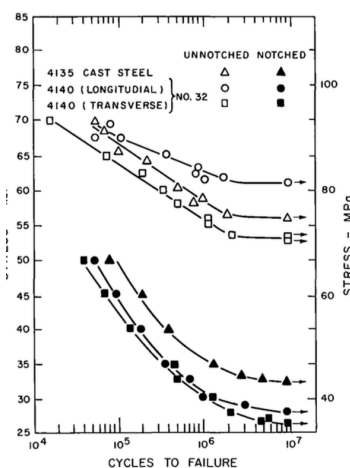
Fatigue is quite complex to model using FEA. In order to simplify things, I ran the fatigue analysis considering only the 500 N bending load, since it's the only load of the three that would be alternating in fatigue. Below is a life plot showing the life in cycles of every element over the surface of the spindle.



**Figure 16:** Fatigue study life plot

As you can see, the study tells us that the spindle has **infinite life** under this loading condition, since I defined infinite life as  $10^7$  cycles. This result is not surprising, since we calculated a high analytical FOS which considered compression and torsion in addition to just bending. However, since I'm not too sure how fatigue simulations really work, I wouldn't trust this value.

Since it required an S-N curve to run, I found one in *Atlas of Fatigue Curves* for notched 4041 steel undergoing a transverse load, which should be appropriate.



**Figure 17:** S-N Curves of 4041 Steel

## Summary

In summary, the analytical and FEA analyses revealed that, even under relatively extreme loading conditions, the spindle maintains a reasonable factor of safety. For the analytical static analysis, both the Rankine and von Mises criteria gave a factor of safety of 3.7. The analytical fatigue analysis, on the other hand, gave a factor of safety of around 10, owing to the smaller forces at play in a fatigue scenario. This suggests that the designer should potentially focus on static failure, instead of fatigue.

The FEA analysis resulted in a factor of safety for the static condition of 1.7, and an infinite factor of safety for fatigue, although this value is questionable.

Though the factors of safety of the static analytical and static FEA differed by a reasonable amount, they agreed on the location of the maximum stress element – the transition from the smaller diameter section to the medium diameter section. Since both the analytical and FEA analyses agree on this point, special attention should be paid to decreasing the stress concentration in that area, perhaps most easily by simply enlarging the fillet.

## Discussion

### Belt Drive Discussion - Cardi Garcia Mendez

In conclusion, the analysis of the drill press mechanism, with a specific focus on the belt drive components, has provided valuable insights into the system's performance and safety considerations. Failure analysis of the belt and chain was conducted through the use of FEA simulations and calculations of Torque, Tight Side Tension (F1), Loose Side Tension (F2), and the resulting safety factor of 1.5.

There were disparities between the FEA and calculations however. The difference in numbers can be because of the assumptions that had to be made to complete the analysis of the belt and chain. For example, for the SolidWorks FEA the material of Nylon101 was used to run the simulation as Nylon is a plastic commonly used for belt drive mechanisms made out of plastic. The material used in the calculations came from table A-4 however it was Polyamide which is made up of some Nylon alloys, but has a different specific weight, allowed tension, and friction factor than a belt made of a Nylon101.

### Spindle Discussion - Alan Deutsch

The key takeaway from this treatment of the drill press spindle is that, shown through both analytical and FEA analysis, the spindle can handle extremely severe misuse with a decent factor of safety in both static and fatigue conditions. Looking at each analysis individually, we predicted a 3.7 factor of safety for the static analytical analysis, 1.7 for static FEA, around 10 for fatigue analytical, and a technically infinite factor of safety for fatigue FEA.

Several trade-offs were made to simplify my analysis, more so for the analytical than FEA. For one, I assumed the spindle has a solid cross section, when in reality it is mostly hollow. This is acceptable, however, because bending and torsion stresses are most significant on the outer surfaces of a part. For the analytical analysis I didn't consider the stress concentrations due to the splines and slotted hole, although the FEA showed the element of maximum stress indeed occurred at point E.

Deviation between the predicted/FEA and theoretical/analytical results can be explained by the analytical calculations not factoring in the various intricacies of the geometry, as mentioned above, and in differences between how the forces and reactions are considered. For example, the analytical calculations consider the bearing reactions and torsion, bending, and compression forces as point forces, whereas the FEA considers them as forces distributed over faces. The FEA can also model the complex interactions between these forces and geometries to a much greater degree of semblance to reality than is possible with purely analytical techniques.

ME41 Project 2  
Alan Deutsch  
Ricardo (Cardi) Garcia Mendez

## Contributions

Cardi Garcia Mendez and Alan Deutsch contributed equally to the executive summary and introduction sections. All belt analysis was conducted by Cardi and all spindle analysis was conducted by Alan.